SOME BASICS IN MODELING OF MECHATRONIC SYSTEMS

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Contents

Summary

This chapter presents some basic concepts for the modeling of lumped-parameter mechatronic systems. Thereby, we think of mechatronics to be the result of a synergetic integration of mechanical engineering with electronics, information and control theory. Thus, by modeling we not only mean the process of setting up the equations for simulation purposes but also the process of revealing the structural aspects of the system, which, in a further step can be exploited for analyzing the system and for designing a controller. It turns out that this kind of modeling is of particular interest for the design of mechatronic systems where the nonlinear behavior cannot be neglected. We will consider a mechatronic system to be an interconnection of various subsystems and/or system

elements in a network-like structure. Clearly, the subsystems (system elements) may stem from different physical domains. Within the modeling process we have to distinguish between the *constitutive relations* of the system elements and the *interconnectivity constraints* which depend on the way these system elements are joined together in a network structure, also referred to as the topological relationship. Thereby, the multidisciplinary nature of mechatronic systems requires a mathematical formulation of the models that is independent of the physical domain. Furthermore, the theory as being established must not rely on the specific choice of a coordinate system. Therefore, we choose a mathematical formulation, which, at least at an introductory level, provides the basis for a coordinate-free (differential-geometric) description of mechatronic systems.

In this chapter, we will focus on an energy-based approach where the subsystems (system elements) are connected through so-called energy ports and where the various system elements, i.e. energy storage elements, coupling elements and static elements, are classified in terms of the energy flows via their ports. Since the network theory has its historical roots in electrical engineering, we will explain the topological relationships of so-called Kirchhoff networks considering electrical networks, although they also apply to more general physical networks. Thereby, the famous theorem of Tellegen proves to be one of the key-concepts in formalizing the power-conserving interconnection of network-type models. In this context the so-called port-Hamiltonian formulation provides a theoretical framework for a systematic energy-based description of mechatronic systems. Throughout the whole chapter we will apply the theory as presented to different examples from the electrical, mechanical and fluidic domain.

1. Introduction

The word *mechatronics* was created by a Japanese engineer in 1969 to describe systems which combine *mecha*nisms with elec*tronics*. Since then the meaning of mechatronics has been gradually extended and by now mechatronics is much more than just the systemic combination of mechanisms and electronics. Mechatronics stands for the philosophy of the synergetic integration of mechanical engineering with electronics, information and control theory. In this sense mechatronics turns out to be a new paradigm for the development and design of intelligent products and new manufacturing processes.

The modeling of mechatronic systems plays an important role in the development process of a mechatronic product. Generally, a model is required for simulation purposes, for analyzing the system and for designing a controller. It is well known that it is rather difficult to set up a model that is appropriate to satisfy all these different demands at the same time. In addition, the multidisciplinary nature of mechatronics brings about that the modeling and control of mechatronic systems require the knowledge of different engineering disciplines. This is why in the last years much effort was put on the development of a unified framework for the modeling and control of (nonlinear) multi-domain physical systems. However, the first works on this field go back to the 1950s and 1960s, where on the one hand the powerful methods of network-based modeling of electrical circuits were extended to other physical domains (mechanical, fluidic, thermal, etc.) and, on the other hand the well established variational methods from mechanics, like the Lagrangian and Hamiltonian formulation, were generalized to nonlinear electrical networks. Basically all the current modeling approaches for

mechatronic systems are based on a *network representation* of interconnected subsystems (system elements) which may stem from different physical domains. Due to the inherent modularity of this concept the models can be easily organized in an object-oriented environment. Furthermore, this modeling approach supports all kinds of top-down and bottom-up design strategies.

It is well known that the *topological relationship* of a network structure can be efficiently described by means of *graph theory*. In general, the way how the various subsystems (system elements) are interconnected plays a central role within the modeling process. The traditional block-diagram oriented approach, where different blocks are connected via arrows, automatically implies certain (computational) causality assumptions on how information is exchanged between the blocks.

Thereby, the signal associated to the arrow is set as the output of one block and serves as the input of another block. In other words, the signal associated to the arrow is an *effect* of the block where it comes from and a *cause* for the block where it is supplied to. If we think of the constitutive law of a linear electrical resistor, it is not clear from the start whether the current is the input and the voltage the output or vice versa.

Thus, in block-diagrams causality assumptions have to be made at the system element level rather than the overall system level and this limits the reusability of the system element models and the possibility of making configuration changes to these models. A suitable concept which can be used to handle these problems is based on an interconnection of subsystems (system elements) via so-called *energy ports*. The subsystem can be thought of as operating on a pair of variables, the so-called *power variables*, whose product is power and the interconnections, the energy ports, are places where power can flow from one subsystem (system element) to another.

A graphical language which optimally supports this kind of physical modeling is given by the so-called *bond-graphs* (see *Modeling and Simulation of Dynamic Systems Using Bond Graphs*). Apart from the bond-graph approach different textual description languages are available for a unified object-oriented modeling of complex physical systems, like Modelica or VHDL-AMS to mention only two important representatives. It is not the intention of this chapter to discuss the terminologies and the concepts of the different modeling languages of mechatronic systems.

In fact, we will rather focus on elaborating some essential principles, which, among others, form the theoretical basis for a systematic modeling of mechatronic systems within a (coordinate-free) mathematical framework which allows us to create a link to results in modern nonlinear control theory. For the sake of clarity we will restrict our investigations to lumped-parameter systems. However, most of the concepts being presented can be extended to the infinite-dimensional case, although their exact mathematical formulation requires many additional technicalities.

2. System Variables and System Elements

Let us consider a mechatronic system composed of different subsystems (system elements) which are interconnected via so-called *energy ports*, see Figure 1.

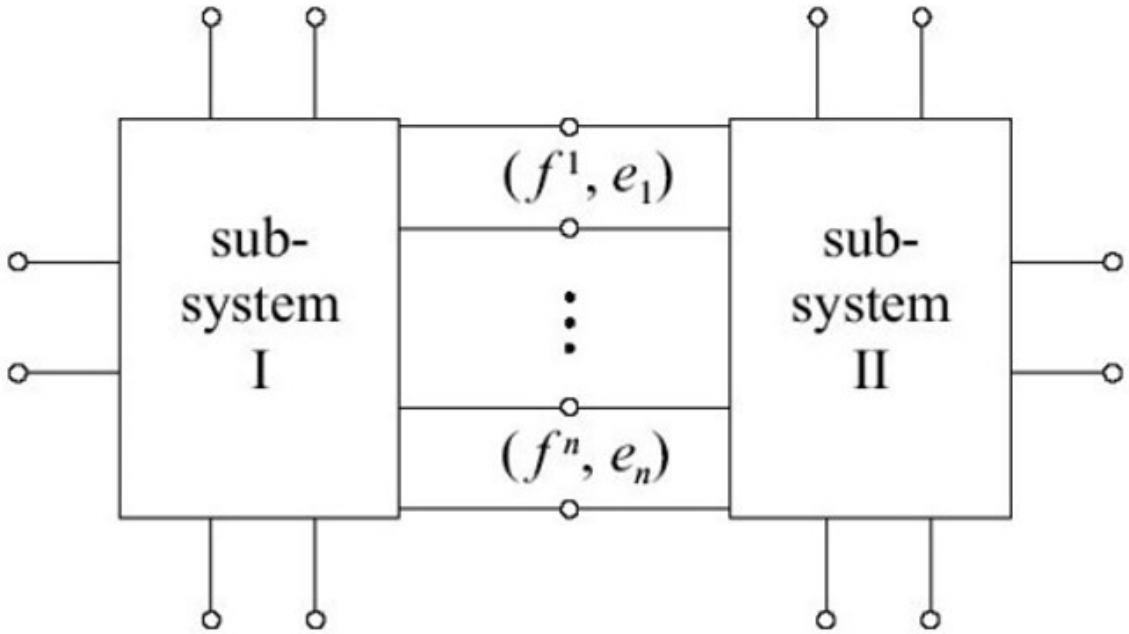

Figure 1. Interconnection of subsystems via energy ports.

An energy port can be represented by a pair of terminals together with a pair of *power variables*, the *efforts* (or *across variables*) e and the *flows* (or *through variables*) f, which describe the energy transfer via the interconnection. Geometrically, the flow variables f are coordinates of a linear vector space $\mathcal{V}$ and the effort variables e are coordinates of the dual vector space $\mathcal{V}^*$ such that the linear map e acting on f, denoted by the duality product $\langle e, f \rangle$, gives the power P flowing via the port from one to the other subsystem.

Subsequently, a component of an effort variable will always have a lower index and a component of a flow variable an upper index, respectively. For simplifying the notation, we will arrange both the efforts e and the flows f in the same way as column vectors and we will write the duality product in the form $P = e^T f = \sum_j e_j f^j$. But bear in mind that in general the distinction between elements of $\mathcal{V}$ and $\mathcal{V}^*$ is essential, since they transform differently under coordinate changes. The currents and voltages in the electrical domain, the velocities and forces (angular velocities and torques) in the pure translational (rotational) mechanical domain as well as the mass flow and the enthalpy for fluidic systems in case of isentropic storage processes serve as appropriate power variables.

2.1. Energy Storage Elements

Next, we will classify energy storage elements like inductors, masses, pressurized fluid tanks etc. For this, consider an ideal energy storage element with an energy port and associated power variables e and f as shown in Figure 2.

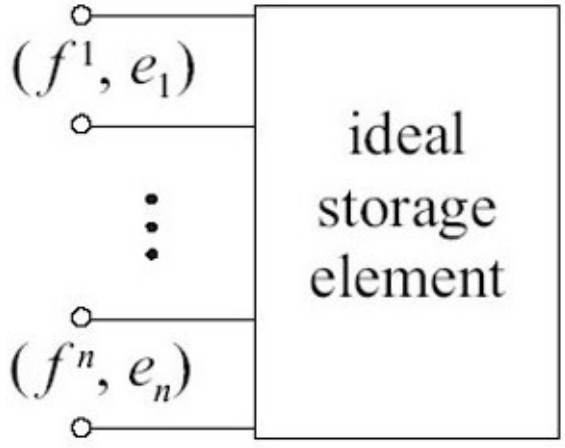

Figure 2: Ideal energy storage element.

Let $P(\tau)$ denote the instantaneous power associated to the port at the time τ. Then the energy transferred over the port in the time interval $[0,t]$ is given by

$$E = \int_0^t P(\tau)\,\mathrm{d}\tau = \int_\gamma e^T f\,\mathrm{d}\tau\,, \tag{1}$$

with γ as a solution curve (the power variables as functions of time) of the system in the time interval $[0,t]$. Clearly, if the energy storage element is ideal, the change of the energy stored in the element is solely determined by the energy transferred over the port. However, we will distinguish between two fundamental mechanisms for the storage of energy, namely the p- and the q-*type storage elements*. For this purpose we introduce two new variables p and q, the so-called *energy variables*, given by

$$p(t) = p(0) + \int_0^t e(\tau)\,\mathrm{d}\tau \quad \text{or} \quad \frac{\mathrm{d}}{\mathrm{d}t}p = e \tag{2}$$

and

$$q(t) = q(0) + \int_0^t f(\tau)\,\mathrm{d}\tau \quad \text{or} \quad \frac{\mathrm{d}}{\mathrm{d}t}q = f\,. \tag{3}$$

In the literature the vector p is often referred to as the *generalized momentum vector* and q as the *generalized displacement vector*. The reason for the choice of these names will become clear later in this section. From (2) and (3) it can be seen that the energy variables p and q are just *state variables* of a simple dynamical system (integrator) with inputs e and f. Sometimes p and q are also referred to as *effort accumulation* and *flow accumulation variables*. As it is the case for state variables, if we know the values of $p(t)$ and $q(t)$ at any given time $t = t_0$, then, given knowledge of the inputs $e(t)$ and $f(t)$ for $t \geq t_0$, we can calculate all present and future values. Thus, the state variables

contain the necessary information about the history of the energy flow.

2.1.1. Generalized Kinetic Energy

Substituting (2) into (1), we get

$$E = \int_\gamma \sum_j f^j \mathrm{d}p_j \,. \tag{4}$$

For this special type of storage device we have to define a constitutive relationship of the form

$$f = \varphi_f(p). \tag{5}$$

Let us assume for the following that the p_j's are linearly independent. Then the integral of (4) is *path independent* iff the 1-form $\sum_j f^j \mathrm{d}p_j$ is exact, or if in a star-shaped region (due to Poincaré's lemma for differential forms) the 1-form is closed, i.e.,

$$\mathrm{d}\left(\sum_j \varphi_f^j(p)\mathrm{d}p_j\right) = 0 \quad \text{or equivalently} \quad \frac{\partial}{\partial p_k}\varphi_f^j(p) = \frac{\partial}{\partial p_j}\varphi_f^k(p) \text{ for all } j \neq k. \tag{6}$$

In the literature the conditions (6) are also sometimes called *integrability conditions* or *Maxwell's reciprocity conditions*. If (6) holds then the stored energy is solely a function of p, given by the expression

$$T(p) = \int^p \sum_j \varphi_f^j(\tilde{p})\mathrm{d}\tilde{p}_j \,. \tag{7}$$

Note that the lower limit of integration for the energy function need not be specified. But in general, it is assumed that the energy is zero at the origin. We will call $T(p)$ a *generalized kinetic energy*. See the mechanical example below for an explanation of this choice of notation. Since T is a function of the energy variable p, this type of storage element is also known as a *p-type storage element* or in bond-graph notation an *I-field*. It can be immediately seen from (7) and (5) that the flow f can be calculated by means of the generalized kinetic energy in the form

$$f^j = \varphi_f^j(p) = \frac{\partial}{\partial p_j}T(p). \tag{8}$$

Following the considerations of Appendix A, we can describe the system in a *contact bundle* with coordinates (p, T, f) and a *contact ideal* generated by the contact 1-form

$$\omega_1 = \mathrm{d}T - \sum_j f^j \mathrm{d}\, p_j \,. \tag{9}$$

The *Legendre transformation* (see Appendix A) to a contact bundle, with new coordinates $\left(\bar{p}, \bar{T}, \bar{f}\right) = \left(f, \sum_j f^j p_j - T, p\right)$ and a contact ideal generated by the 1-form

$$\omega_2 = \mathrm{d}\bar{T} - \sum_j \bar{p}_j \mathrm{d}\, \bar{f}^j \,, \tag{10}$$

leads to the so-called *generalized kinetic co-energy* function

$$\bar{T}(f) = \int^f \sum_j \left(\varphi_f^{-1}\left(\tilde{f}\right)\right)_j \mathrm{d}\, \tilde{f}^j \,, \tag{11}$$

provided that the function $\varphi_f(p)$ in (5) is invertible. Note that this is certainly the case if $T(p)$ is strictly convex.

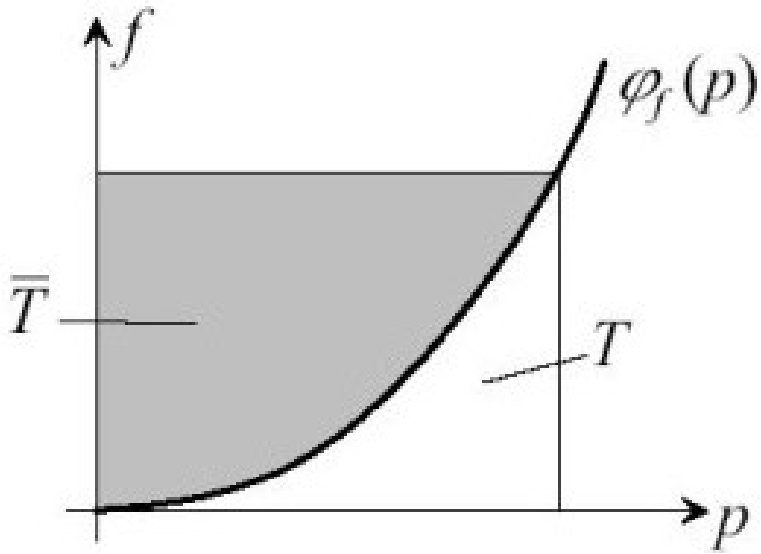

Figure 3. Graphic illustration of the kinetic energy and co-energy function for a one-port p -type storage element.

Furthermore, condition (6) implies the relation $\mathrm{d}\left(\left(\varphi_f^{-1}(f)\right)_j \mathrm{d}f^j\right) = 0$, which ensures the path independence of the integral in (11). Analogous to (8) we can directly calculate p from the kinetic co-energy

$$p_j = \left(\varphi_f^{-1}(f)\right)_j = \frac{\partial}{\partial f^j}\bar{T}(f) \,. \tag{12}$$

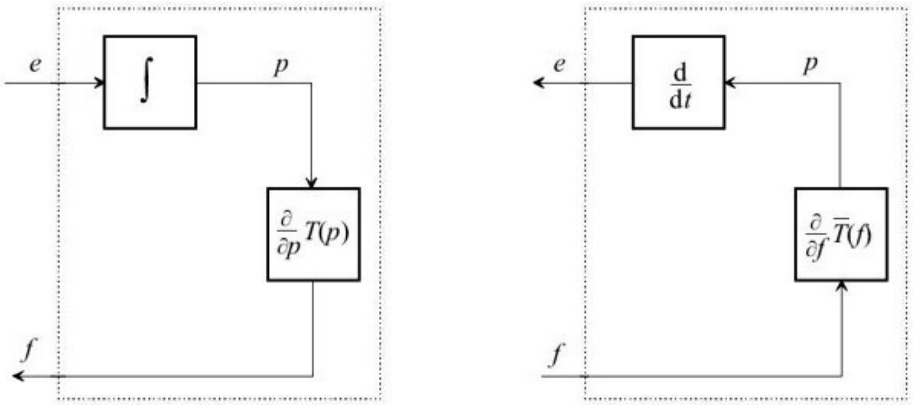

Figure 4: Causal representations of a p -type storage element.

A graphic illustration of the kinetic energy and co-energy for a one-port p -type storage element with a nonlinear constitutive law (5) is depicted in Figure 3. Combining (2), (8) and (12) we have two causal representations of the p -type storage element depending on whether the effort e is the input and the flow f the output or vice versa. Figure 4 shows these representations in the form of two block-diagrams. In bond-graph terminology the representation on the left side of Figure 4 is called *integral causality* and the one on the right side *derivative causality* referring to the presence of an integrator and a derivative block element, respectively (see *Modeling and Simulation of Dynamic Systems Using Bond Graphs* for the different sorts of causality assignment in a bond-graph). In this context it is worth mentioning that the energy variable serves as a state variable and hence the energy is a function of the state whereas the co-energy is not.

Mechanical Example – Single Translation Mass: Consider a single translational mass m with the velocity v (flow f) and the force ε (effort e) as the power variables. It is well known that the energy variable p of (2) corresponds to the momentum p because of the relation $\frac{\mathrm{d}}{\mathrm{d}t} p = \varepsilon$. The constitutive relationship (5) between the velocity v and the momentum p is given by $v = m^{-1} p$. Thus, the (generalized) kinetic energy due to (7) follows to

$$T(p) = \int^{p} m^{-1} \tilde{p} \mathrm{d}\tilde{p} = \frac{1}{2} m^{-1} p^2 \tag{13}$$

and the (generalized) kinetic co-energy takes the form, see (11),

$$\bar{T}(v) = \int^{v} m \tilde{v} \mathrm{d}\tilde{v} = \frac{1}{2} m v^2 \,. \tag{14}$$

Note that the expression of (14) is usually referred to as the kinetic energy of the mass m. Properly spoken, it is a co-energy because it is a p -type storage element and thus is expressed as a function of the flow variable. This example should also clarify the names

generalized kinetic energy for $T(p)$ and generalized momentum for the energy variable p.

Electrical Example – Mutually Coupled Inductors: Given an energy storage element consisting of n mutually coupled inductors with associated power variables, current $i = \left(i^1,\ldots,i^n\right)$ (flow f) and voltage $u = \left(u_1,\ldots,u_n\right)$ (effort e), see Figure 5.

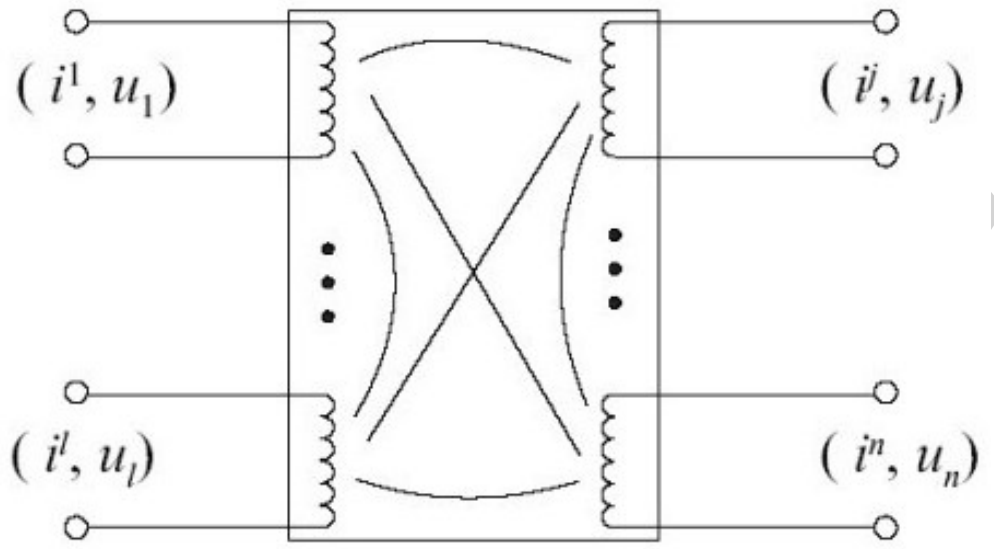

Figure 5: Mutually coupled inductors as an energy storage element.

The voltage u is related to the flux linkage $\psi = \left(\psi_1,\ldots,\psi_n\right)$ by the differential equation $\frac{\mathrm{d}}{\mathrm{d}t}\psi = u$. Hence the flux linkage ψ corresponds to the energy variable (generalized momentum vector) p of (2). With the constitutive relation $i = \varphi_f\left(\psi\right)$ due to (5) we can calculate the (generalized kinetic) energy and co-energy in the form

$$T(\psi) = \int^{\psi} \sum_{j=1}^{n} \varphi_f^j\left(\tilde{\psi}\right)\mathrm{d}\tilde{\psi}_j \quad \text{and} \quad \bar{T}(i) = \int^{i} \sum_{j=1}^{n} \left(\varphi_f^{-1}\left(\tilde{\imath}\right)\right)_j \mathrm{d}\tilde{\imath}^{\,j}\,. \tag{15}$$

The condition for the path-independence of (11) can be interpreted as the symmetry condition of the so-called *incremental inductance matrix* $L(i) = \left[L_{jk}(i)\right] = \left[\partial\left(\varphi_f^{-1}(i)\right)_j / \partial i^k\right]$. In the case when the matrix L is constant the energy and co-energy functions of (15) simplify to

$$T\left(\psi\right) = \frac{1}{2}\psi^T L^{-1}\psi \quad \text{and} \quad \bar{T}\left(i\right) = \frac{1}{2} i^T L i\,. \tag{16}$$

Note that in this context the demand on $T(\psi)$ or $\bar{T}(i)$ to be strictly convex is equivalent to the demand on L to be positive definite.

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Biographical Sketch

Andreas Kugi was born in Villach, Austria, in 1967. He received his Dipl.-Ing. degree in Electrical Engineering from the Technical University of Graz, Austria, and his Ph.D. (Dr.techn.) degree in Control Engineering from the Johannes Kepler University, Linz, Austria, in 1992 and 1995, respectively.

In 1992, he joined the Department of Automatic Control and Control Systems Technology at the Johannes Kepler University of Linz, where he worked as an assistant in research and education from 1992 to 1995, as an Assistant Professor from 1995 to 2000 and as an Associate Professor from 2000 to 2002. He received his "Habilitation" degree in the field of Automatic Control and Control Theory from the Johannes Kepler University of Linz in 2000. In 2002, he was appointed Full Professor at the Saarland University, Saarbrücken, Germany, where he currently holds the Chair of System Theory and Automatic Control.

The research interests of Andreas Kugi include the physics-based modeling and control of nonlinear mechatronic systems; differential geometric and algebraic methods for nonlinear control with applications to underactuated mechanical systems and hydraulic servo-systems; and infinite-dimensional control concepts with applications to smart structures and MEMS. He is author of the monograph "Non-linear Control Based on Physical Models" (London: Springer, LNCIS 260, 2001) and he has published more than 100 research papers in international scientific journals and conference proceedings volumes. For his scientific contributions Andreas Kugi was awarded the Kardinal Innitzer Science Prize in 2001, Vienna, Austria.

Andreas Kugi is involved in several industrial research projects in the field of automotive applications, rolling mills, hydraulic servo-drives, control of smart materials and the design and optimization of MEMS in cooperation with national and international companies. More than 10 patents result from these activities.